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Numerical thermomechanical modelling of shape memory alloy wires

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Abstract

The dynamic properties of straight shape memory alloy (SMA)-actuator wires are characterised by the rate of heating and cooling procedures. Based on an analysis of energy fluxes into and out of the actuator, a numerical model that simulates the time response of SMA-wires is presented. Cause variables that have an impact on the actuator behaviour (actuator geometry, provided load, arbitrary time variant heating current, environmental conditions) can easily be implemented in the model. Taking these changing boundary conditions into consideration, the model enables a simple design layout for technical solutions with straight SMA-wires.

Keywords: SMA-wires; SMA-actuator; Time response; Activation; Numerical model; Thermomechanical modelling

1. Introduction

Though a generally accepted definition or standardisation of the term 'actuator' is still missing, the following specification is widely respected: actuators are always a series circuit of an energy provider and an energy transducer. An actuator is called 'new actuator' when its function is essentially based on physical characteristics of new transducer materials (e.g. shape memory alloys, SMAs) [1]. In order to use SMAs as a switching transducer material, the actuator has to be activated either by external heating devices or by electric current using the ohmic resistance (e.g. of SMA-wires). For the realisation of cyclic movements, external resetting forces (constant load, bias spring) have to be applied to the SMA-actuator during cooling (extrinsic two-way-effect). For various reasons this option is preferred for design and industrial applications based on SMAs. The results of the research presented in this paper are all based on the use of binary nickel titanium (NiTi) as SM-alloy and on the use of the extrinsic two-way-effect.

The dynamic behaviour of the transducer material is of great interest for actuator applications. Therefore, the simulation of the SMA-actuator time response is highly reasonable as it facilitates and supports the design layout process. Though analytic models for the characterisation of the SMA's time response exist already (e.g. [2]), these methods are strongly simplified in order to enable their solvability. Furthermore, these models do not allow the consideration of variable heating currents and variable loads (e.g. bias springs).

2. Thermodynamics and energy balance

The activation of SMAs is solely specified by the velocity of cooling and heating the material. Many factors determine the dynamic behaviour of SMA-actuators: the chemical alloy composition, the actuator geometry, the transformation temperatures (TT; M_f, M_s, A_s, A_f), the position and width of the hysteresis, the electrical heating current (dc) I and its cycle $I(t)$, the mechanical load as well as the environmental conditions. Especially with time variable current profiles and variable load conditions, these parameters cannot or only inadequately be considered by simplifying algebraic functions. In order to characterise the SMA actuator system and its activation behaviour, all time variant and invariant energy fluxes that pass the system border have to be

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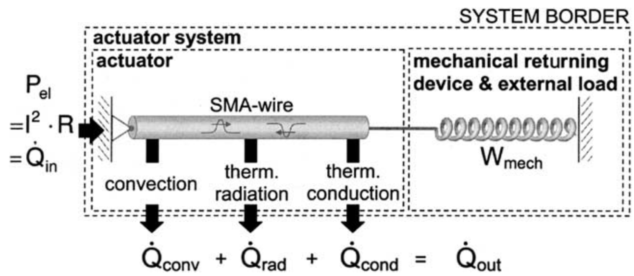

Fig. 1. Energy balance of the SMA-wire–actuator system.

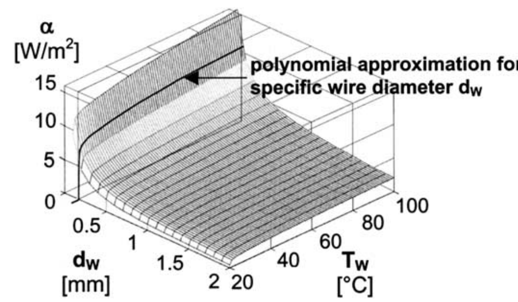

Fig. 2. Convective heat-transfer coefficient $\alpha = f(T_W, d_W)$.

balanced. This balance is prepared for a straight SMA-wire in Fig. 1.

The only energy source during the heating phase is an external electric current that heats the wire by the use of its ohmic resistance R_{el}. Regarding the heat transmission for the cooling process of the actuator, three effects are distinguished in thermodynamics: convection, thermal radiation and thermal conduction [3]. The heat transmission caused by free air convection is about 90% and by heat radiation about 10% of the total emission. The low influence of heat conduction due to the clamping of the wire can be neglected. Comparable to the consideration of the emitted heat flow, the required mechanical energy resulting from the returning device and external loads as well as the integral latent heat for transformation from martensite to austenite have to be quantified.

The required latent heat for transformation is proportional to the derivation of the volumetric martensite fraction ξ. Its value for the applied wires was specified by a differential scanning calorimetry (DSC)-measurement to about 24,000 J/kg. According to the first law of thermodynamics, the caloric state equation for the actuator inner energy results in the following relations for heating and cooling.

Heating:

$$c_p \rho V_W \frac{dT_W}{dt} + \alpha F_W (T_W - T_\infty) + \varepsilon_m \sigma_{rad} F_W ({T_W}^4 - {T_\infty}^4) + \rho V_W \Delta H \left| \frac{d\xi}{dt} \right| + \frac{dW_{mech}}{dt} = I^2(t) R_{el}$$

Cooling:

$$c_p \rho V_W \frac{dT_W}{dt} + \alpha F_W (T_W - T_\infty) + \varepsilon_m \sigma_{rad} F_W ({T_W}^4 - {T_\infty}^4) - \rho V_W \Delta H \left| \frac{d\xi}{dt} \right| - \frac{dW_{mech}}{dt} = 0$$

where c_p is the specific heat capacity, F_W the actuator surface, I the heating current, R_{el} the ohmic resistance, T the ambient temperature, T_W the wire temperature, V_W the actuator volume, W_{mech} the mechanical energy, ΔH the integral latent heat for transformation, α the convective heat-transfer coefficient, ε_m the emission ratio, ρ the actuator density, σ_{rad} the Stephan–Bolzman constant, ξ the volumetric martensite fraction.

These non-linear first order differential equations cannot be solved with analytic methods, which means that the solution term of T_W cannot be defined in form of an equation. Thus, the solution curve will be calculated numerically by using an approximation procedure. Besides the solvability of the problem, the main advantages of using a numerical method is the possibility of implementing time variable parameters that depend on the mixture between martensite and austenite and thus have an impact on the phase transformation. Thereby the quality of the generated model can be enhanced. These parameters are as follows:

Convective heat-transfer coefficient α: The heat-transfer coefficient α is the characteristic parameter for convective heat-transfer and visualisable as a 3D surface plot depending on the wire-temperature T_W and diameter d_W (Fig. 2). The numerical model is based on a polynomial approximation of α for each examined wire-diameter.

Influence of stress on the transformation temperatures: The TT depend on the current mechanical stress inside the wire. According to the modified Clausius–Clapeyron equation the TT are in linear proportion to the actuator mechanical stress condition. Using bias springs the actuator stress condition is not constant, so the phase transformation martensite–austenite is not only thermal but also stress-induced. For this reason M_f, M_s, A_s and A_f have to be computed iteratively in the numerical model using the current mechanical stress in the actuator wire.

Thermal expansion ε_{therm}: Besides the contraction of the actuator in consequence of the shape memory effect, an opposite thermal expansion of the actuator has to be considered. The different coefficients of thermal expansion of martensite and austenite (β_M, β_A) have to be taken into account.

$$\varepsilon_{therm} = (T_W - T_\infty)(\xi \beta_M + (1 - \xi)\beta_A)$$

Elastic elongation ε_{elast}: The conventional elastic elongation ε_{elast} of the wire actuator depends, apart from the actuator geometry and of course the stress, basically on the actuator structure. The different Young's moduli of martensite and austenite and the phase mixture during transformation have to be considered [4].

$$\varepsilon_{elast} = \sigma \left(\frac{\xi}{E_M} + \frac{1 - \xi}{E_A} \right)$$

Ohmic resistance R_{el}: The wire's ohmic resistance corresponds to the ratio of martensite (ρ_M) and austenite (ρ_A) during phase transformation and to the current actuator geometry.

$$R_{el} = \frac{l_W}{A_W}(\xi\rho_M + (1-\xi)\rho_A)$$

Geometry: When contracting, a variation of the SMA-wire's geometry (length, diameter, surface) is induced. Provided that the actuators volume is constant, these values are determined in real-time during simulation and linked to the calculation of other process parameters (e.g. the ohmic resistance).

For the analytic description of the SMA's characteristics during phase transformation an interconnection with a hysteresis model that has to be specified by the input of the actuator attributes is necessary [5]. For this purpose a simple hysteresis model from Liang and Rogers [6] that segmentally defines the characteristic hysteresis curve of a SMA with trigonometric functions in the different temperature ranges, is implemented into the numerical model.

3. Numerical model

The computer aided implementation of the differential equations and the relevant process parameters in order to simulate the time response of a straight SMA-actuator wire during phase transformation is realised in MATLAB/SIMULINK®. This fact ensures a necessary universality of the model. The complete SMA-actuator system is built in the shape of a block diagram that consists of data sources and sinks, algebraic functions, logic operations and customised subsystems. Important inputs for the SIMULINK®-model (e.g. length and diameter of the actuator wire, constant of the opposed bias spring(s), environmental conditions) are separately combined in a MATLAB-script for easy specification. Fig. 3 shows a small extract of the SIMULINK®-block diagram for the heating phase.

By using virtual switches (left) the heating current can be defined (constant value, sine wave, ramp, arbitrary sequence, etc.). The energy sinks (convection, radiation, latent heat for transformation, mechanical energy) are defined in subsystems and added to the model. In the loop shown, the wire temperature, the mechanical stress, the transformation temperatures depending on the stress and the martensite fraction are calculated.

A comparable block diagram which can be specified by the input of the initial values for the start temperature T_W-start and the start martensite fraction ξ-start was generated for the cooling phase.

4. Results

Depending on the defined boundary conditions and the heating current, the stress depending transformation temperatures (M_f, M_s, A_s, A_f) and the wire temperature T_W are

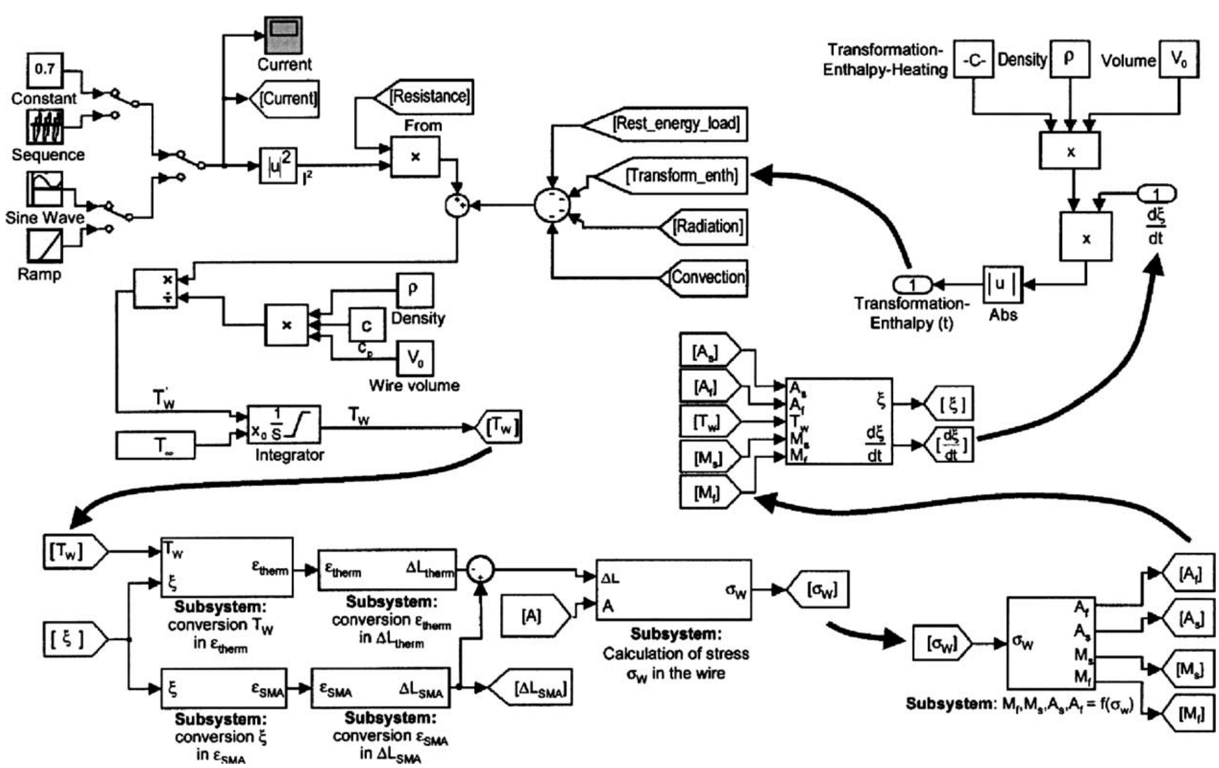

Fig. 3. MATLAB/SIMULINK-model: heating phase.

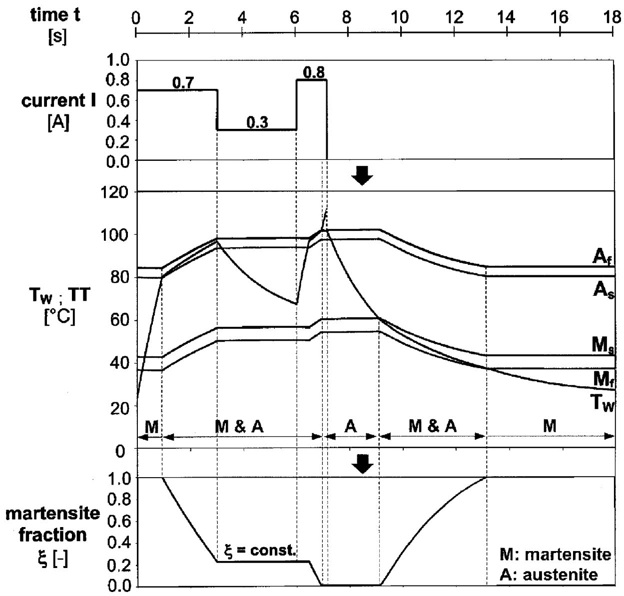

Fig. 4. Simulation results for heating and cooling.

calculated. Using the hysteresis model the current martensite fraction ξ during phase transformation is computed. Some simulation results for a variable run of the heating current and a following cooling phase can be seen in Fig. 4. The initial material properties of the used binary NiTi-alloy in a state free of stress are: $M_f = 28\,°C$; $M_s = 34.5\,°C$; $A_s = 71.5\,°C$; $A_f = 76\,°C$; actuator length: 800 mm; Ø-actuator: 0.254 mm. The spring constant of the used bias spring is $c = 327$ N/m.

As soon as the wire temperature equals the austenite-start-temperature (A_s), the phase transformation starts, ξ decreases as the material begins to change its phase to austenite and the actuator contracts (Fig. 4). As a consequence, the mechanical stress inside the wire increases due to the fact that the bias spring is stretched. Therefore, the transformation temperatures increase too. When the current is reduced, the wire temperature decreases also but the martensite fraction is constant as T_W is still above the martensite-start-temperature (M_s). When increasing the current, T_W increases until austenite-finish (A_f) and beyond. During cooling ($I = 0$) the wire temperature reaches M_s and the transformation to martensite begins. When reaching martensite-finish (M_f), ξ is again equal to 1, as the actuator is in fully martensite phase.

Depending on the martensite fraction ξ the current stroke of the actuator Δl_W is calculated. First simulation results of the actuator time response, when heated with variable (ramp and steps) and constant electric currents, were validated experimentally. An experimental setup for analysing the time response of straight SMA-actuator wires has been developed for this purpose (Fig. 5).

The SMA-wire (6) is clamped between a force transducer (7) on the one side and a linear motion slide (3) that is connected with the bias spring (1) and a displacement transducer (5) on the other side. The programmable power source enables the generation of a totally arbitrary heating current that is switched over to the actuator with a relay.

In order to validate the SIMULINK®-model a straight SMA-wire is analysed experimentally and compared numerically. For simulation and experiment, the applied load, the heating current profile and the boundary conditions were of course the same.

The following figures confirm the physically good implementation of the actuator properties and its behaviour in the SIMULINK®-model (Fig. 6).

It is obvious that the computed stroke–time curves match the measured curves well. Differences between the measured and the simulated process can only be detected at the beginning of the transformation. These differences may occur as a result of simplified simulation boundary conditions (e.g. constant surrounding temperature in the near actuator proximity; homogeneous state of stress in the actuator though the polycrystalline structure of the wire leads to scaling effects).

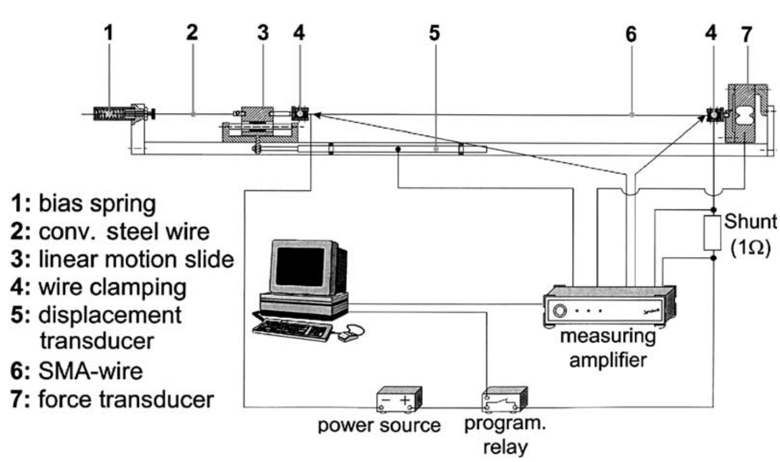

Fig. 5. Experimental set-up.

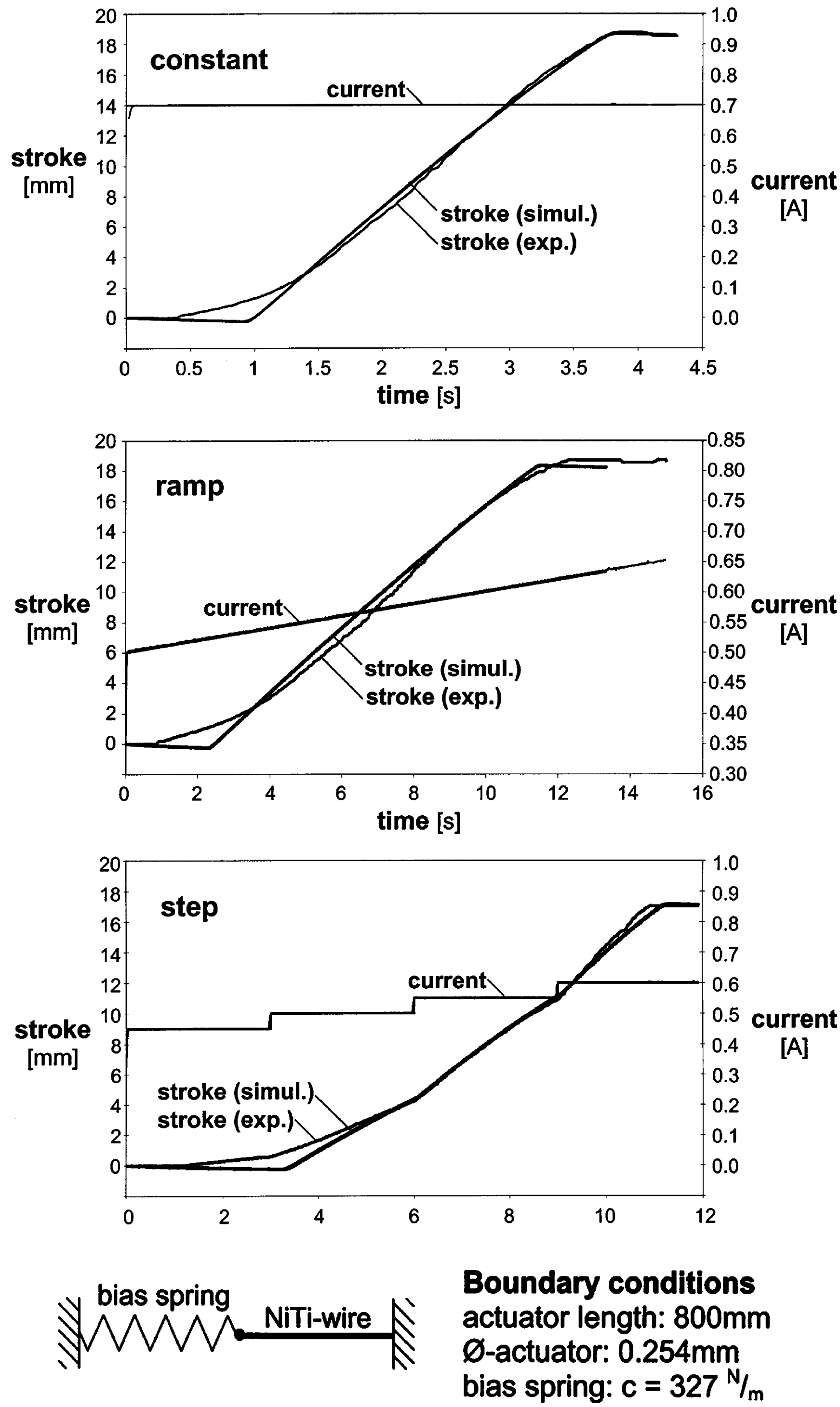

Fig. 6. Simulation vs. experiment: results.

5. Conclusion

Based on an energy balance of the actuator system and under consideration of important cause variables that have an impact on the martensite–austenite transformation, a numerical model for simulating the activation behaviour of SMA-actuator wires was presented. The model is characterised by its simple possibility to be specified with the relevant SMA characteristics and it is therefore suited for the design layout of technical solutions with SMA-actuator wires. It allows to consider different environmental conditions, variable heating currents as well as different time variable loads. The qualitatively good implementation of the system properties was experimentally proven. In the future it has to be examined how the model can be extended and adapted to other SMA-actuator designs (e.g. beams, thin films, etc.) with various modifications.

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